Rules vs. Discretion in Authority[1]

Susheng Wang[2]

October 2019

Abstract: An organization faces a stream of random events everyday. Whether to set managerial rules or allow managerial discretion is a key issue in organizations. In this paper, we investigate the boundaries between rules and discretion in authority. Using an incomplete contract approach, we differentiate between projects that are more efficiently managed under rules and those that are more efficiently managed under discretion. Our main finding is that for conservative projects with low expenditures and balanced expenditure to quality ratios, rules are more efficient than discretion; for other projects, discretion is efficient. We also find that (1) rules offer better incentives; (2) discretion works better for risky projects; (3) whenever discretion is efficient, rules are equally efficient; and (4) the income share of the manager is independent of her decision-making rights, i.e., separation of income and control rights.

Keywords: managerial rules, managerial discretion, organizations, incomplete contracts

JEL classification: M16, L23

[1] An earlier version of this paper is entitled “Rules vs. Discretion in Organizations”, circulated in July 2016.

[2] Address: Hong Kong University of Science and Technology. Email: s.wang@ust.hk.

1. Introduction

Every authority faces many matters each day. Which matters should be dealt with under rules and which matters should be dealt with under discretion? Every organization has its own set of rules and hierarchy of authority. Both are needed for an organization to run efficiently. In practice, decision-making in large organizations tend to be under rules, while decision-making in small organization tend to be under discretion. A key purpose of having a hierarchy of authority is to facilitate discretionary decisions. Discretion means freedom to make decisions. In a time horizon, rules mean decision-making ex ante, while discretion means decision-making ex post. A firm anticipating opportunistic projects in the future will set out rules and discretion in the firm's organizational structure. Some projects are better handled under rules, while other projects are better decided under discretion. The purpose of this paper is to offer a theory on rules vs. discretion in a firm's decision-making. We use an incomplete contract approach, by which the contract to a manager includes a set of decision-making rights. To our knowledge, such an approach has never been applied to this issue before.

One possible justification for discretion is that rules are incomplete due to transaction costs. A manager has to decide for herself when rules do not mention what to do. However, we will not rely on this simple justification. Instead, we will show that even when rules can be set without transaction costs, they may be intentionally set incomplete so that discretion can serve to improve efficiency under certain circumstances. Under a complete contract, a manager is obliged to follow the rules specified in the contract; under an incomplete contract, a manager has the discretion to make ex post decisions based on the circumstances. For a firm anticipating opportunistic projects in the future, taking into account risks and incentive problems, what projects are better decided under rules and what projects are better decided under discretion? Our study addresses this question by comparing the advantage of rules in offering incentives with the advantage of discretion in handling risks.

Managerial decisions in practice are made based on either rules or discretion. Hiring by certain criteria such as age, working experience or educational attainment is an example of decision-making under rules. Rules are popular in public bureaucracies, suggesting potential benefits from rules. At the same time, public bureaucracies are also known for red tape, implying negative effects of rules on social welfare. Discretion offers flexibility when changes or unexpected events occur. At the same time, discretion encourages decision makers' opportunistic behaviors. In practice, conservative projects with low expenditures or low expenditure to quality ratios tend to be decided under rules. For example, firms often follow given rules when hiring low-level labor since it involves low salaries and its value to the firm is relatively clear. Also, every company has some small discretionary budget that managers can tap into when needed. On the other hand, discretion often applies to risky projects with high expenditures or high expenditure to quality ratios. For example, the hiring of high-level labor is often based on discretion since it involves high salaries and its value to the firm varies across individuals. Most employees have well-defined responsibilities that give them little freedom in decision-making, since conservative projects with low expenditures and balanced expenditure to quality ratios are common. However, company executives are typically given much freedom in decision-making. Executives encounter a wide range of projects and face many unexpected situations involving supplies, competitors, distributors and other stakeholders. In an environment of great uncertainty, executives need freedom in decision-making in order to handle future projects efficiently. Discretion allows executives to tap their experiences, expertise, judgement and wisdom to make the right decisions. Krug (2009) quoted Carl von Clausewitz's view that the outcome of direct conflict is largely unpredictable, since "people often interpret the same events differently and competitors often respond in unanticipated ways". Hence, executives need discretion to respond to an ever-changing business environment effectively. Indeed, our theory is consistent with these observations. Although a firm may derive huge benefit from discretion, there is little research on the boundaries between a manager's discretion and rules. The purpose of this study is to identify these boundaries by comparing the effects of decision-making under rules with those under discretion given different circumstances.

In government policymaking, rules vs. discretion is a well-known debate. Time consistency is the key issue. However, our focus is on organization of production. The key issue is the allocation of decision-making rights. The assignment of decision-making rights to a manager has an effect not only on incentives but also on how effectively risks are handled. The disadvantage of rules is clear: rules are set with hypothetical scenarios in mind. The advantage of rules is that they offer clear policy on how to deal with events. That is, rules are bad for handling risks but good for incentives. On the other hand, discretion has the advantage of postponing decision-making until uncertainty is resolved. Risks are better handled ex post when uncertainty is resolved. The disadvantage of discretion is that the manager will hesitate to invest effort since discretionary decisions are made after such investment is sunk. That is, discretion is better for handing risks but bad for incentives. In a time horizon, discretion focuses on ex post efficiency, while rules focus on ex ante efficiency. The question is which projects are better handled under rules and which projects are better decided under discretion.

There is an extensive literature on managerial discretion. Many studies are based on the argument that discretion results in agency costs. Williamson (1964) proposed a model of managerial discretion, in which the manager has control over discretionary investment. With a separation of ownership and control, the manager can use discretion to design and execute policies that would maximize her own utility rather than that of shareholders. James (1983), Schiff & Weisbrod (1991), and Bises (2000) described the negative effects of discretion on efficiency and effectiveness of the organizational activities of non-profit firms. Milgrom (1988) and Milgrom & Roberts (1992, 2009) suggested that discretion may have distributive effects, which cause the agents working in the firm to undertake "influence activities" with possible negative effects on the firm's productivity. Milgrom (1988) described a benevolent principal who maximizes a social objective defined by a weighted sum of the firm's profit and the employee's utility. Aghion & Tirole (1997) discuss under what conditions the principal in a principal-agent model delegates authority to the agent. In contrast, by introducing a principal who cares about fairness into Milgrom's (1988) model, Antonelli (2014) showed that discretion will always result in an improvement of the firm's performance. Antonelli (2003) showed that, when distributive effects are endogenous (the effects are included in total organizational output), influence activities are self-limited. Spreitzer & Porath (2012) pointed out that "employees at every level are energized by the ability to make decisions that affect their work. Empowering them in this way gives them a greater sense of control, more say in how things get done, and more opportunities for learning." Discretion in our model reflects these benefits. Crossan (2005) surveyed a few alternative theories to the standard theory of the firm, by which discretionary behaviors can be explained. Using an incomplete contract approach, we find that both discretion and rules have important roles to play. In the literature, managerial discretion has been associated with managerial abuse of power. One common solution is to use rules to limit discretion. Different from prior literature, we completely ignore the possibility of managerial corruption or abuse of power from discretion. Instead, we focus on discretion's advantage in risk handling vs. rules' advantage in offering incentives.

Neither the empirical nor the theoretical literature on managerial discretion to date has conclusively resolved the well-known so-called discretion puzzle, namely whether additional discretion can increase, decrease or maintain performance.[3] Furthermore, for the last 20 years, we have seen an increasingly important role of rules in organizations. This trend limits managerial discretion. Our study aims to resolve this discretion puzzle. We find that the effect of discretion on performance is dependent on the circumstances, so that additional discretion increases, decreases or maintains performance depending on the circumstances.

In contrast to extensive discussion on the effects of discretion in prior literature, the effects of rules have rarely been discussed. We investigate the relative efficiency of rules vs. discretion under various circumstances. Our model is unique in that we allow both advantages and disadvantages of rules and discretion and find out when it is better to use discretion over rules and vice versa. That is, we identify the boundaries between rules and discretion in a firm's organizational structure. We are the first to use an incomplete contract approach in solving this problem: we treat rules as a complete contract and discretion as an incomplete contract. Our contract specifies a profit-sharing scheme and decision-making rights; that is,

[3] See Agarwal et al. (2009), Barnabas & Mekoth (2010), Chang & Wong (2003), Gammelgaard et al. (2010), Khanchel (2009), Caza (2011), Groves et al. (1994), Li & Zhao (2004), López-Navarro & Camisón-Zornoza (2003), Venaik (1999), He et al. (2009), Heinecke (2011), Stano (1976), Williamson (1963), and Xu et al. (2005).

our contract is broad enough to contain both income and control rights. Our result does not one-sidedly favor rules or discretion. Instead, we differentiate between those projects that are better handled under rules and those that are better handled under discretion. This result is consistent with practice, in which organizations typically impose rules on certain matters while allowing discretion on other matters. Different from Milgrom (1988), Milgrom & Roberts (1992, 2009) and Antonelli (2003, 2014), our model does not consider influence activities. Instead, we consider the advantage of rules in offering incentives versus the advantage of discretion in controlling risks.

It makes sense to devise a broad contract specifying both a profit-sharing scheme and an allocation of decision-making rights, since most managers care about both income and control rights, and these two rights may be dependent on each other. We derive both decision-making rights (Proposition 4) as well as income rights in equilibrium (Proposition 5). On the boundaries between rules and discretion, our main finding is that for conservative projects with low expenditures and balanced expenditure to quality ratios, rules are more efficient than discretion; for other projects, discretion is efficient. We also find that rules offer better incentives; discretion works better for risky projects; and whenever discretion is efficient, rules are equally efficient. Whatever the decision-making rights are, the profit-sharing scheme is a linear scheme of the form $s(\pi) = \alpha + \beta\pi$, where π is the firm's profit, $s(\pi)$ is the manager's income, and α and β are constants. We find that the profit share β is independent of decision-making rights, implying a separation of income and control rights as far as rules and discretion are concerned.

In prior literature, when discussing control rights, income rights are often ignored. In our model, both income and control rights are endogenously determined in equilibrium. The key here is that we allow endogenous dependence of income and control rights. This is important since income and control rights may be dependent on each other in equilibrium. For example, a manager may be willing to accept a lower pay for greater discretion. Equity joint ventures and contractual joint ventures are such examples (Wang & Zhu, 2005). Our model is the first to include both a profit-sharing scheme and decision-making rights in a contract on this topic. This model makes our theory applicable to many applications. For example, we can compare the allocations of control rights between equity joint ventures and contractual joint ventures. We can also compare the decision-making rights between managers of hedge funds and managers of mutual funds. In hedge funds, investors link managerial pay to performance by requiring a co-investment by the manager. The hedge fund approach is the same as our approach in a discretionary contract that includes a profit-sharing scheme to the manager. In contrast, in mutual funds, investors rely on regulation, disclosure and risk limits. The mutual fund approach is the same as our approach in a rules-based contract that sets rules on investment decisions. Agarwal et al. (2015) carried out an empirical study on these two types of funds recently. Their findings support a positive relationship between performance and the requirement of co-investment by the manager in hedge funds. Our theory supports their empirical findings on discretionary contracts in hedge funds. However, our theory also supports the rules-based approach in mutual funds, although Agarwal et al. (2015) did not find such evidence. Depending on the nature of investment, both rules and discretion can be efficient.

This paper proceeds as follows. In Section 2, we set up the model. In Section 3, in a general setting, we derive the optimal solution under rules and the optimal solution under discretion. In Section 4, in a parametric setting, we study various circumstances under which rules may be inferior or superior to discretion. In Section 5, we conclude with some remarks. All the proofs and derivations are shown in the Appendix.

2. The Model

Timing

An organization faces a stream of random events everyday. Some events must be dealt with under rules while others should be dealt with under discretion. More specifically, a firm randomly encounters investment opportunities. A principal representing the firm hires a manager to manage investment. The principal can be a higher-ranked manager or the owners as a group. The principal may give the manager discretionary decision-making rights or specify decision-making rules.

The relationship lasts two periods and yields profit π at the end (date 2). At the beginning, the manager invests an effort $a \in \mathbb{A}$ and the principal simultaneously invests an effort $b \in \mathbb{B}$, with private costs $c(a)$ and $C(b)$, respectively, where $\mathbb{A}$ and $\mathbb{B}$ are respectively the spaces for a and b. The two efforts generate a joint investment $h(a,b)$, where h is a real-valued function. The profit is random ex ante with density function $f[\pi|h(a,b)]$ conditional on joint effort $h(a,b)$. The timing of the events is shown in Figure 1.

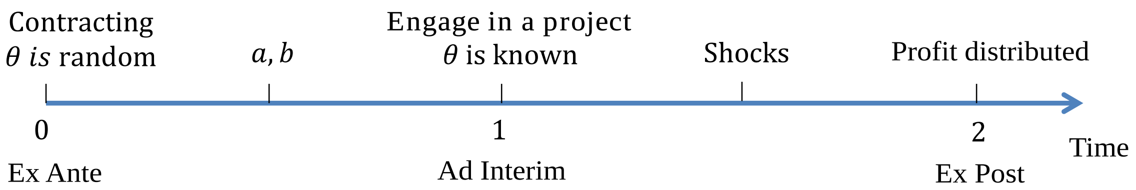

Figure 1. The Timing of Events

Contracts

The principal hires and signs a contract with the manager. In practice, a managerial contract contains not only a salary package but also a set of decision-making rights. Hence, our contract contains two components: a salary package and a decision-making right. Assume that profit π is verifiable ex post. The salary package is a scheme for sharing profit π and is denoted by $s(\pi)$. More precisely, $s(\pi)$ is the portion of π paid to the manager. Denote the set of admissible salary schemes by

$$\mathbb{S} = \{s: \mathbb{R} \to \mathbb{R} \mid s \text{ is Lebesgue integrable}\}.$$

We will show that, among these admissible contracts, a linear scheme of the form $s(\pi) = \alpha + \beta\pi$ is optimal, where α and β are constants and $\beta > 0$. Linear profit-sharing schemes are popular in practice.

In addition to a salary scheme, the contract also offers a decision-making right. The principal can specify rules of engagement in future projects or let the manager decide on the spot. A contract is called a rules-based contract if the principal specifies rules on future projects in the contract; otherwise, it is called a discretionary contract. More specifically,

$$\text{A rules-based contract} = \{s(\pi), \text{ rules for the manager to follow}\},$$
$$\text{A discretionary contract} = \{s(\pi)\},$$

where $s(\pi)$ is the compensation for the manager's work. Our focus is on rules versus discretion. In the discretionary contract, no rules are specified, and hence the manager is free to make decisions based on her own judgement. The first contract a complete contract and the second an incomplete contract.

Efforts a and b are observable ad interim but not verifiable so they are not specified in a contract. Since efforts are made after a contract is accepted, incentives must be provided to induce high efforts. The two contracts have different effects on efforts. Since a decision to engage in a project is made after efforts are sunk, discretion generally has a negative effect on efforts. Hence, incentives are affected not only by the profit-sharing scheme but also by the decision-making right. Our task is to compare these two contracts so as to gain an understanding of whether to adopt rules or discretion in organizations.

Payoffs

Assume that both the principal and the manager are risk neutral in income. Hence, the principal's and the manager's ex post payoffs are respectively

$$\text{the manager's payoff:} \quad s(\pi) - c(a)$$
$$\text{the principal's payoff:} \quad \pi - s(\pi) - C(b).$$

Since the ex ante payoffs are concave in efforts, both the principal and the manager are risk averse in efforts, and risk matters.

Projects

A project has a quality index $\theta \in \mathbb{R}$ representing its intrinsic value to the firm. The project can be an internal reorganization of the firm, an M&A, a divestiture, an emerging investment opportunity, or an expansion or contraction of product lines. This quality index may represent synergy when the firm acquires another firm or an improvement in productivity or efficiency when the firm divests a division. Without the project, the firm's density function of profit is $f[\pi|h(a,b)]$; with the project, the density function becomes $f[\pi|h(a,b),\theta]$. Quality index θ is random ex ante, but becomes known ad interim. Assume that information about θ is symmetric, such that the two parties have common distribution and density functions $\Phi(\theta)$ and $\phi(\theta)$ respectively for θ.

Denote the <u>interim expected profits</u> at $t = 1$ by

$$\bar{\pi}(a,b,\theta) \equiv \int_{\mathbb{R}} \pi f[\pi|h(a,b),\theta]d\pi, \qquad \bar{\pi}(a,b) \equiv \int_{\mathbb{R}} \pi f[\pi|h(a,b)]d\pi.$$

The costs of efforts have already been sunk at this point in time. Standard assumptions are imposed on the functions as follows: $\bar{\pi}(a,b)$ and $\bar{\pi}(a,b,\theta)$ are increasing and concave in a and b separately, $c(a)$ and $C(b)$ are positive, increasing and convex, and $\bar{\pi}(a,b,\theta)$ is also increasing in θ. The <u>interim gain from the project</u> is

$$g(a,b,\theta) \equiv \bar{\pi}(a,b,\theta) - \bar{\pi}(a,b).$$

If the manager invests in the project based on the quality index θ being larger than a threshold θ_t, then the <u>ex ante gain from the project</u> is

$$\bar{g}(a,b,\theta_t) \equiv \int_{\theta_t}^{\infty} g(a,b,\theta)d\Phi(\theta). \tag{1}$$

Then, the <u>ex ante profit</u> after taking into account the gain from the project is

$$\Pi(a,b,\theta_t) \equiv \bar{\pi}(a,b) + \bar{g}(a,b,\theta_t).$$

Driving Forces

There are two forces driving the results from our model. First, under rules, since investment decisions are specified in the contract, efforts are made only after investment decisions are made. In contrast, under discretion, since investment decisions are not specified in the contract, efforts are made and sunk before investment decisions are made, implying less incentive to apply efforts. Hence, rules are good for incentives. Second, under rules, investment decisions are made before uncertainty about investment opportunities is realized. In contrast, under discretion, investment decisions are made after uncertainty about investment opportunities is realized, so that investment can adjust to realized opportunities. Hence, discretion is good for handling risks.

3. Two Management Regimes in a General Setting

In this section, we set up and solve for the two contractual solutions.

3.1. Managerial Discretion

Consider the case in which the principal offers a discretionary contract that gives the manager the freedom to make decisions on future projects. In a discretionary contract, θ need not be verifiable but it has to be observable ad interim.

Given efforts a and b, after θ is known, the manager will invest in a project with quality θ ad interim if and only if $g(a,b,\theta) \geq 0$. Define $\theta_d(a,b)$ by the following equation:

$$\bar{\pi}(a,b,\theta_d) = \bar{\pi}(a,b), \tag{2}$$

where the subscript d stands for "discretion". Then, if and only if $\theta \geq \theta_d$ will the manager invest in the project. In other words, θ_d is the threshold above which the manager will invest in the project. Then, the ex ante gain from the project is $\bar{g}[a,b,\theta_d(a,b)]$, and the ex ante profit for the firm is $\Pi[a,b,\theta_d(a,b)]$. The manager's ex ante income is

$$U(a,b,s) \equiv \int_0^{\theta_d(a,b)} \int s(\pi) f[\pi|h(a,b)] d\pi d\Phi(\theta) + \int_{\theta_d(a,b)}^{\infty} \int s(\pi) f[\pi|h(a,b),\theta] d\pi d\Phi(\theta).$$

The principal's ex ante income is

$$\begin{aligned} V(a,b,s) &\equiv \int_0^{\theta_d(a,b)} \int [\pi - s(\pi)] f[\pi|h(a,b)] d\pi d\Phi(\theta) \\ &\quad + \int_{\theta_d(a,b)}^{\infty} \int [\pi - s(\pi)] f[\pi|h(a,b),\theta] d\pi d\Phi(\theta) \\ &= \int_0^{\theta_d(a,b)} \bar{\pi}(a,b) d\Phi(\theta) + \int_{\theta_d(a,b)}^{\infty} \bar{\pi}(a,b,\theta) d\Phi(\theta) - U(a,b,s) \\ &= \bar{\pi}(a,b) + \int_{\theta_d(a,b)}^{\infty} g(a,b,\theta) d\Phi(\theta) - U(a,b,s) \\ &= \Pi[a,b,\theta_d(a,b)] - U(a,b,s). \end{aligned} \tag{3}$$

After the contract is signed, the two parties choose efforts a and b simultanously in a Nash game. The Nash equilibrium of (a,b) is determined by

$$U_a(a,b,s) = c'(a), \qquad V_b(a,b,s) = C'(b),$$

where U_a and V_b are respecitively the partial derivatives of U and V with respect to a and b. These two conditions are called incentive compatibility (IC) conditions. Similarly, denote by Π_a and Π_b the partial derivatives of Π with respect to a and b, respectively. Then, the principal's problem is

$$
\begin{aligned}
W_d^* \equiv \max_{s\in\mathbb{S},a\in\mathbb{A},b\in\mathbb{B}} & \Pi[a,b,\theta_d(a,b)] - U(a,b,s) - C(b) \\
\text{s.t.} \quad & IC_a{:}\, U_a(a,b,s) = c'(a), \\
& IC_b{:}\, V_b(a,b,s) = C'(b), \\
& IR{:}\;\; U(a,b,s) \geq c(a),
\end{aligned}
\tag{4}
$$

where the last condition is the individual rationality (IR) condition for the manager. Its solution is presented in the following proposition and its proof is shown in the Appendix.

Proposition 1 (Discretion). *Under discretion, the equilibrium efforts* (a_d^*, b_d^*) *are determined by*

$$
\begin{aligned}
W_d^* = \max_{a\in\mathbb{A},b\in\mathbb{B}} & \Pi[a,b,\theta_d(a,b)] - c(a) - C(b) \\
\text{s.t.} \quad & \Pi_a[a,b,\theta_d(a,b)] = c'(a) + \frac{h_a(a,b)}{h_b(a,b)} C'(b).
\end{aligned}
\tag{5}
$$

And, an optimal profit-sharing scheme is linear:

$$
s(\pi) = c(a_d^*) + \frac{c'(a_d^*)}{\Pi_a[a_d^*, b_d^*, \theta_d(a_d^*, b_d^*)]}\{\pi - \Pi[a_d^*, b_d^*, \theta_d(a_d^*, b_d^*)]\}. \quad \blacksquare
$$

The linear contract has two components: a monetary transfer and a fixed profit share. The monetary transfer can be paid upfront and thus has no effect on incentives. It is the fixed profit share that offers incentives to the manager. Interestingly a fixed profit share is sufficient for the purpose. This linear sharing scheme is consistent with reality, where linear contracts are popular.

Remark. One advantage of a linear scheme is that the validity of the first-order approach, i.e., $U_{aa}(a,b,s^*) < c''(a)$ and $V_{bb}(a,b,s^*) < C''(b)$ for all (a,b), can be guaranteed by requiring concavity of the profit functions $\bar{\pi}(a,b)$ and $\bar{\pi}(a,b,\theta)$ in (a,b).

3.2. Managerial Rules

Consider the case in which the principal offers a rules-based contract that specifies rules for the manager to follow. In a rules-based contract, θ needs to be verifiable ad interim. By this, the principal can state explicitly in the contract that the manager must invest in the project if and only if the quality index is above a threshold.

Let θ_r be the threshold above which the manager must invest in the project, where the subscript r stands for "rules". Since θ_r is verifiable, this minimum quality standard can be written into a contract and is enforceable by law. Then, the ex ante gain and profit are $\bar{g}(a,b,\theta_r)$ and $\Pi(a,b,\theta_r)$, respectively. The manager's ex ante income is

$$U(a,b,\theta_r,s) = \int_0^{\theta_r}\int s(\pi)f[\pi|h(a,b)]d\pi d\Phi(\theta) + \int_{\theta_r}^{\infty}\int s(\pi)f[\pi|h(a,b),\theta]d\pi d\Phi(\theta),$$

and the principal's ex ante income is

$$\begin{aligned} V(a,b,\theta_r,s) &\equiv \int_0^{\theta_r}\int [\pi - s(\pi)]f[\pi|h(a,b)]d\pi d\Phi(\theta) \\ &\quad + \int_{\theta_r}^{\infty}\int [\pi - s(\pi)]f[\pi|h(a,b),\theta]d\pi d\Phi(\theta) \\ &= \int_0^{\theta_r}\bar{\pi}(a,b)d\Phi(\theta) + \int_{\theta_r}^{\infty}\bar{\pi}(a,b,\theta)d\Phi(\theta) - U(a,b,\theta_r,s) \\ &= \bar{\pi}(a,b) + \int_{\theta_r}^{\infty} g(a,b,\theta)d\Phi(\theta) - U(a,b,\theta_r,s) \\ &= \Pi(a,b,\theta_r) - U(a,b,\theta_r,s). \end{aligned} \tag{6}$$

After the contract is signed, the two parties choose efforts a and b simultanoursly in a Nash game. The Nash equilibrium of (a,b) is determined by

$$U_a(a,b,\theta_r,s) = c'(a), \qquad V_b(a,b,\theta_r,s) = C'(b).$$

Then, the principal's problem is

$$\begin{aligned} W_r^* \equiv \max_{s\in\mathbb{S},a\in\mathbb{A},b\in\mathbb{B},\theta_r\geq 0} & \ \Pi(a,b,\theta_r) - U(a,b,\theta_r,s) - C(b) \\ \text{s.t.} \qquad & IC_a: U_a(a,b,\theta_r,s) = c'(a), \\ & IC_b: V_b(a,b,\theta_r,s) = C'(b), \\ & IR:\ \ U(a,b,\theta_r,s) \geq c(a). \end{aligned} \tag{7}$$

Its solution is presented in the following proposition and its proof is shown in the Appendix.

Proposition 2 (Rules). *Under rules, the equilibrium efforts and the quality threshold* $(a_r^*, b_r^*, \theta_r^*)$ *are determined by*

$$\begin{aligned} W_r^* \equiv \max_{a\in\mathbb{A},b\in\mathbb{B},\theta_r\geq 0} & \ \Pi(a,b,\theta_r) - c(a) - C(b) \\ \text{s.t.} \qquad & \Pi_a(a,b,\theta_r) = c'(a) + \frac{h_a(a,b)}{h_b(a,b)}C'(b). \end{aligned} \tag{8}$$

And, an optimal profit-sharing scheme is linear:

$$s(\pi) = c(a_r^*) + \frac{c'(a_r^*)}{\Pi_a(a_r^*,b_r^*,\theta_c^*)}\left[\pi - \Pi(a_r^*,b_r^*,\theta_r^*)\right]. \ \blacksquare$$

4. Rules vs. Discretion in a Parametric Setting

We first consider a benchmark case in which there are no incentive problems.

Proposition 3. *If there are no incentive problems, rules and discretion are equally efficient.*

Proposition 3 states that rules and discretion are equally efficient if there are no incentive problems. Hence, risk alone cannot drive a wedge between rules and discretion; incentives play an important role in the boundaries between rules and discretion. To compare the two management options in detail, we use parametric functions and represent a few key aspects using parameters. We have arbitrarily chosen the following set of functions:

$$\begin{aligned} &h(a,b) = a^{\mu_1} b^{\mu_2}, \qquad c(a) = \gamma_1 a, \qquad C(b) = \gamma_2 b, \\ &\bar{\pi}(a,b) = h(a,b), \qquad \bar{\pi}(a,b,\theta) = (1+\theta)h(a,b) - I, \end{aligned} \tag{9}$$

where θ represents an improvement in productivity from a project, $\gamma_1 > 0$ and $\gamma_2 > 0$ are the marginal costs of effort, $\mu_1 > 0$ and $\mu_2 > 0$ represent the marginal contributions of the two parties with $\mu_1 + \mu_2 < 1$, and I is the project expenditure. Assume that θ takes two possible values: $\theta = 0$ and $\theta = \theta_0 > 0$ with θ_0 occurring with probability p. That is, there is a project that may improve productivity by amount θ_0 with probabilty p for investment I.[4] Among the parameters, μ_i and γ_i are individual parameters representing the characteristics of the two parties, and θ_i, p and I are environmental parameters representing the characteristics of the project.

The following proposition contains our main result. Its proof, including the derivation of the solution, the configuration of Figure 2 and the definition of (a^*_{d-}, b^*_{d-}), (a^*_{d+}, b^*_{d+}) and Λ, is shown in the Appendix.

Proposition 4 (Rules vs. Discretion). *Given the parametric specification in (9),*

(a) As indicated in Figure 2, for conservative projects in zone E, rules are efficient; for projects in other zones, discretion is efficient.

(b) Whenever discretion is efficient, rules are equally efficient.

(c) In all zones, $a^*_d \le a^*_r$ and $b^*_d \le b^*_r$, indicating better incentives under rules.

(d) For risky projects in zone D, discretion is efficient, indicating better handling of risks under discretion.

---

[4] This expenditure can be random ex ante. A random expenditure will not affect our results.

In Figure 2, three curves divide the space into five zones. We have designated zones A to D as zones for discretion. Our model implies that rules and discretion are equally efficient in these zones. However, if we take into account possible transaction costs associated with setting and enforcing rules, discretion is better in these zones.

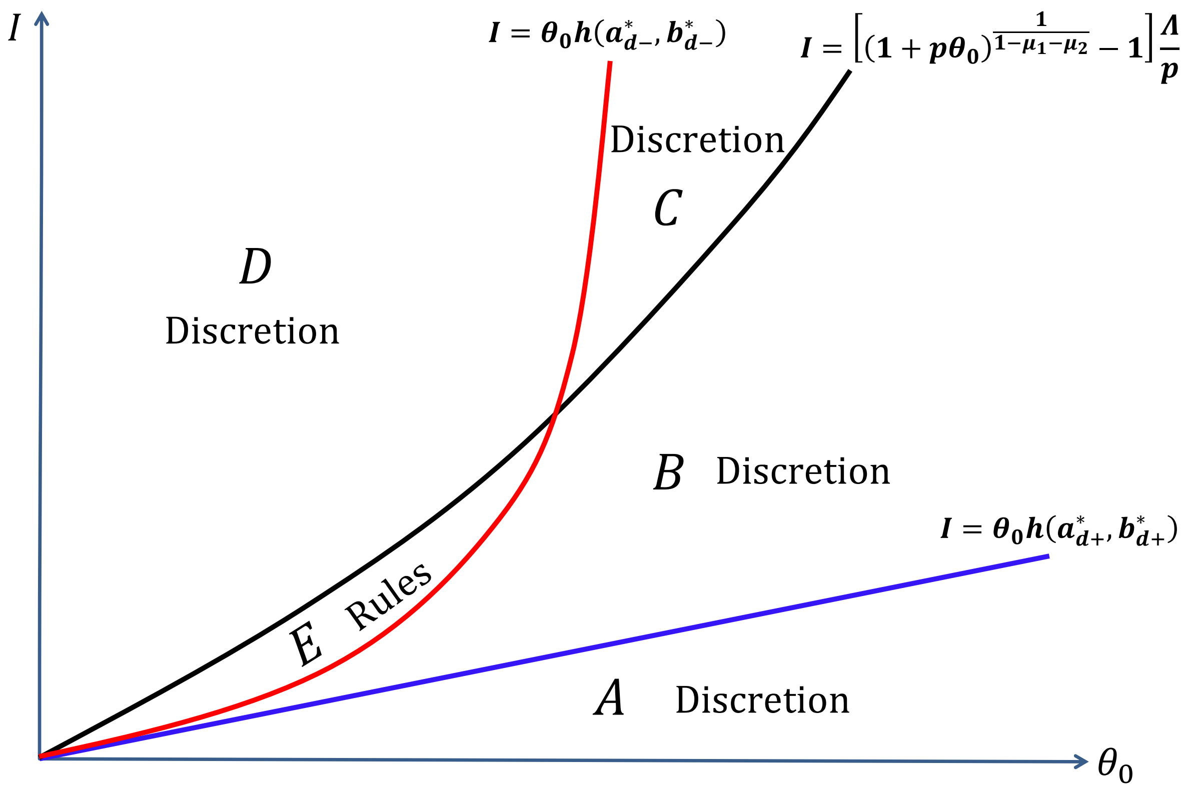

Figure 2. Rules vs. Discretion

For convenience, projects in zone A are called A projects, projects in zone B are called B projects, and so on. Clearly, A projects are excellent projects, with low expenditure/quality (I/θ_0) ratios. B projects are average projects, with reasonable expenditure/quality ratios. When p increases, zone B expands and zone D contracts. C projects are high potential projects, called growth projects, with high expenditures but reasonable expenditure/quality ratios. D projects are risky projects, with high expenditure/quality ratios. When the marginal costs of efforts increase, zone A contracts and zone D expands. Finally, E projects are conservative projects, with low expenditures and balanced expenditure/quality ratios.

E projects are special. Our results indicate that for conservative projects, rules are strictly better than discretion. Discretion has an advantage in dealing with risky projects, but this advantage has no role to play for conservative projects. In contrast, rules have an advantage in offering better incentives, which has a role to play for conservative projects. This explains why rules are more efficient than discretion for conservative projects.

D projects are also special. Although rules offer better incentives, the main issue for these projects is the risk. In contrast, with its advantage in dealing with risk, discretion suits risky projects. Indeed, our theory indicates that discretion is efficient for dealing with risky projects. Kuratko et al. (2014) observe that "entrepreneurial opportunities are often best recognized by those with discretion over how to perform their work, as well as by those encouraged to engage in experimentation". This observation is consistent with our result that discretion allows a manager to handle unexpected projects better.

Our results support rules. Rules are efficient for all projects. For E projects, rules are strictly better than discretion. For other projects, rules are as efficient as discretion. At the same time, our results also support discretion. Except for the conservative E projects, discretion is efficient. If we take into account transaction costs in setting and enforcing rules, discretion is better for most projects except for the conservative projects.

In practice, rules typically apply to those matters and projects that can be clearly defined. The explanation lies in the low transaction costs that such matters and projects incur. For example, age, working experience and educational attainment are often used in hiring rules, since they can be clearly defined and easily verified. But when a project is complicated and hard to define, setting and enforcing rules for it will incur high transaction costs. If in that, discretion is better.

Proposition 4(b) indicates that for most projects, rules and discretion are equally efficient. This is consistent with the fact that rules and discretion coexist in all organizations.

Propositions 4(a) and 4(b) indicate that, if there are no transaction costs in setting and enforcing rules, rules are efficient for all projects. Hence, for projects with low transaction costs, rules may be adopted in practice. For example, in developed countries where institutions are well established, rules are popular; while in developing countries where institutions are not well established, discretion is popular. The reason is that when institutions are well established, costs have already been incurred to set up institutions for enforcing rules and there are no additional transaction costs to enforce new rules. For example, rules are popular in public bureaucracies which are typically well-established institutions. One implication is that rules become popular after institutions become well established.

Rules encourage investment since the threshold to invest is set before efforts are invested. In contrast, under discretion, the threshold to invest is set after efforts are sunk, which reduces the incentives to invest. This explains why efforts are larger under rules in Proposition 4(c).

Under discretion, the manager can wait until the quality of a project becomes known before deciding to invest. Doing so allows the manager to avoid investing in money-losing projects. This explains why discretion works better for risky projects, as indicated in Proposition 4(d).

Proposition 5 (Separation of Income and Control Rights).

(a) In every zone, the profit shares for the two contracts are the same.

(b) All zones give the same profit share of

$$\beta = \frac{\mu_1 + (1-\mu_2)\sqrt{\dfrac{\mu_1(1-\mu_1)}{\mu_2(1-\mu_2)}}}{1 + \sqrt{\dfrac{\mu_1(1-\mu_1)}{\mu_2(1-\mu_2)}}}. \quad \blacksquare$$

Our incomplete contract is a broad contract specifying a profit-sharing scheme and an allocation of decision-making rights. This makes sense since a manager is expected to care about both income and control rights, and these two rights may be dependent on each other. The income rights are presented in Proposition 5. It turns out that, whatever the decision-making rights are, the profit-sharing scheme is a linear scheme of the form $s(\pi) = \alpha + \beta\pi$, where α and β are constants. This is consistent with the popularity of linear sharing schemes in practice. On the issue of the interdependence of the two rights, we find that the profit share β is independent of decision-making rights; only the one-time transfer α is dependent on decision-making rights. That is, we have a separation of income and control rights, when control rights are limited to rules and discretion.

Proposition 5 offers several implications of the contracts under rules and discretion. First, it suggests that the rules-based and discretionary contracts differ only in an upfront monetary transfer; their profit shares are the same under any circumstances. Second, the profit share to the manager is independent of whether decisions are made under rules or discretion. Third, the profit share is determined only by the two parties' output contributions μ_1 and μ_2; other factors are irrelevant. If $\mu_1 + \mu_2 \to 1$, then $\beta \to \mu_1$, implying that the manager's contribution share μ_1 in output very much determines her profit share.

5. Concluding Remarks

Our theory is applicable to random events in general. Our general conclusion is that authorities differentiate some events to be dealt with under rules and others under discretion. Incentives and risks decide the boundaries separating these two approaches in authority.

Although there is an extensive literature on the effects of discretion, to our knowledge, our work is the first to investigate the boundaries between rules and discretion. We find that some projects are more efficiently handled under rules while other projects are more efficiently handled under discretion, consistent with the observation that rules and discretion coexist in every organization. Our model is unique in that it is based on an incomplete contract approach, in which rules are defined as a complete contract and discretion is defined as an incomplete contract.

In practice, rules have certain flexibility and discretion has certain restrictions. In this paper, for a sharp contrast, we consider two extreme cases: strict rules vs. full discretion. Comparing these two extreme cases offers us a good understanding of the issue. However, for practical applications, we need to allow a degree of flexibility in rules and a degree of constraint in discretion. The issue then becomes flexible rules vs. constrained discretion.

Our model opens up many research possibilities on the topic. There are many other factors that can be considered within our model setting, such as asymmetric information, externalities, responsibilities, diverse interests, and bureaucratic inefficiency. In this study, we ignored the possibility of an abuse of power under discretion as we wanted to focus on the influence of other factors that have not been considered in prior literature. We also did not explicitly include transaction costs under rules, although we did use them to justify the adoption of discretion when rules and discretion are equally efficient. Adding this factor is straightforward.

Appendix

This appendix contains all the proofs.

Proof of Proposition 1

By equation (2) which defines θ_d, we find

$$\frac{\partial \theta_d}{\partial a} = \frac{\bar{\pi}_a(a,b) - \bar{\pi}_a(a,b,\theta_d)}{\bar{\pi}_\theta(a,b,\theta_d)}, \qquad \frac{\partial \theta_d}{\partial b} = \frac{\bar{\pi}_b(a,b,\theta_d) - \bar{\pi}_b(a,b)}{\bar{\pi}_\theta(a,b,\theta_d)}.$$

We have

$$\begin{aligned} \bar{\pi}_a(a,b,\theta) &= h_a(a,b)\int \pi f_h[\pi|h(a,b),\theta]d\pi\,, \\ \bar{\pi}_b(a,b,\theta) &= h_b(a,b)\int \pi f_h[\pi|h(a,b),\theta]d\pi\,, \end{aligned} \tag{10}$$

and similarly for $\bar{\pi}_a(a,b)$ and $\bar{\pi}_b(a,b)$. Then,

$$\frac{1}{h_a(a,b)}\frac{\partial \theta_d(a,b)}{\partial a} = \frac{1}{h_b(a,b)}\frac{\partial \theta_d(a,b)}{\partial b}. \tag{11}$$

Then, by (10), we have

$$\frac{\bar{g}_a(a,b,\theta_d)}{h_a(a,b)} = \frac{\bar{g}_b(a,b,\theta_d)}{h_b(a,b)}. \tag{12}$$

We obviously have

$$\frac{\bar{\pi}_a(a,b)}{h_a(a,b)} = \frac{\bar{\pi}_b(a,b)}{h_b(a,b)}.$$

Thus,

$$\frac{\Pi_a(a,b,\theta_d)}{h_a(a,b)} = \frac{\Pi_b(a,b,\theta_d)}{h_b(a,b)}. \tag{13}$$

Therefore, the following two equations imply each other:

$$\Pi_a(a,b,\theta_d) = c'(a) + \frac{h_a(a,b)}{h_b(a,b)}C'(b), \qquad \Pi_b(a,b,\theta_d) = C'(b) + \frac{h_b(a,b)}{h_a(a,b)}c'(a).$$

Also, according to the definition in (1), we have

$$\bar{g}_\theta(a,b,\theta_d) = -g(a,b,\theta_d)\phi(\theta_d).$$

Since $g[a,b,\theta_d(a,b)] = 0$, we have $\bar{g}_\theta[a,b,\theta_d(a,b)] = 0$. Thus,

$$\frac{d}{da}\bar{g}[a,b,\theta_d(a,b)] = \bar{g}_a[a,b,\theta_d(a,b)] + \bar{g}_\theta[a,b,\theta_d(a,b)]\frac{d\theta_d}{da} = \bar{g}_a[a,b,\theta_d(a,b)],$$

where $\bar{g}_a(a,b,\theta_d)$ is the partial derivative w.r.t. a assuming a fixed θ_d. Thus,

$$\frac{d}{da}\Pi[a,b,\theta_d(a,b)] = \Pi_a[a,b,\theta_d(a,b)]. \tag{14}$$

Symmetrically, this expression also holds for variable b. Further, we have

$$\begin{aligned}
U_a(a,b,s) = h_a \int_0^{\theta_d}\int s(\pi) f_h[\pi|h(a,b)]d\pi d\Phi(\theta) + \phi(\theta_d)\frac{\partial\theta_d}{\partial a}\int s(\pi) f[\pi|h(a,b)]d\pi \\
+h_a \int_{\theta_d}^{\infty}\int s(\pi) f_h[\pi|h(a,b),\theta]d\pi d\Phi(\theta) - \phi(\theta_d)\frac{\partial\theta_d}{\partial a}\int s(\pi) f[\pi|h(a,b),\theta_d]d\pi\,, \\
U_b(a,b,s) = h_b \int_0^{\theta_d}\int s(\pi) f_h[\pi|h(a,b)]d\pi d\Phi(\theta) + \phi(\theta_d)\frac{\partial\theta_d}{\partial b}\int s(\pi) f[\pi|h(a,b)]d\pi \\
+h_b \int_{\theta_d}^{\infty}\int s(\pi) f_h[\pi|h(a,b),\theta]d\pi d\Phi(\theta) - \phi(\theta_d)\frac{\partial\theta_d}{\partial b}\int s(\pi) f[\pi|(a,b),\theta_d]d\pi\,.
\end{aligned}$$

By (11),

$$\frac{U_a(a,b,s)}{h_a(a,b)} = \frac{U_b(a,b,s)}{h_b(a,b)}. \tag{15}$$

Thus, by (3), (13), (14) and (15),

$$\begin{aligned}
V_b(a,b,s) &= \frac{d}{db}\Pi[a,b,\theta_d(a,b)] - U_b(a,b,s) = \Pi_b[a,b,\theta_d(a,b)] - U_b(a,b,s) \\
&= \frac{h_b}{h_a}\Pi_a[a,b,\theta_d(a,b,I)] - \frac{h_b}{h_a}U_a(a,b,s) = \frac{h_b}{h_a}V_a(a,b,s),
\end{aligned} \tag{16}$$

implying

$$U_a(a,b,s) = \Pi_a[a,b,\theta_d(a,b)] - V_a(a,b,s).$$

Therefore, using (16), IC_a and IC_b in the principal's problem (4) imply

$$
\begin{aligned}
c'(a) = U_a(a,b,s) &= \Pi_a[a,b,\theta_n(a,b)] - V_a(a,b,s) \\
&= \Pi_a[a,b,\theta_n(a,b)] - \frac{h_a}{h_b} V_b(a,b,s) = \Pi_a[a,b,\theta_n(a,b)] - \frac{h_a}{h_b} C'(b).
\end{aligned}
\tag{17}
$$

This equation can replace one of the IC conditions. Also, since the IR condition must be binding, using (17), the principal's problem (4) becomes

$$
\begin{aligned}
\max_{s\in\mathbb{S},a\in\mathbb{A},b\in\mathbb{B}} \; & \Pi[a,b,\theta_d(a,b)] - c(a) - C(b) \\
\text{s.t.} \quad & \Pi_a[a,b,\theta_d(a,b)] = c'(a) + \frac{h_a(a,b)}{h_b(a,b)} C'(b), \\
& U_a(a,b,s) = c'(a), \\
& U(a,b,s) = c(a).
\end{aligned}
$$

This problem can be solved in two steps. We first find the optimal efforts (a_d, b_d) from

$$
\begin{aligned}
\max_{a\in\mathbb{A},b\in\mathbb{B}} \; & \Pi[a,b,\theta_d(a,b)] - c(a) - C(b) \\
\text{s.t.} \quad & \Pi_a[a,b,\theta_d(a,b)] = c'(a) + \frac{h_a(a,b)}{h_b(a,b)} C'(b).
\end{aligned}
\tag{18}
$$

Then we find a salary package $s(\pi)$ that satisfies the following two conditions:

$$
U_a(a,b,s) = c'(a), \tag{19}
$$

$$
U(a,b,s) = c(a). \tag{20}
$$

Given efforts (a_d^*, b_d^*) from problem (18), consider a linear scheme of the form $s(\pi) = \alpha + \beta\pi$. Then,

$$
\begin{aligned}
U(a,b,s) &= \alpha + \beta \left\{ \int_0^{\theta_d(a,b)} \int \pi f[\pi|h(a,b)] d\pi d\Phi(\theta) + \int_{\theta_d(a,b)}^{\infty} \int \pi f[\pi|h(a,b),\theta] d\pi d\Phi(\theta) \right\}, \\
&= \alpha + \beta \left\{ \int_0^{\theta_d(a,b)} \bar{\pi}(a,b) d\Phi(\theta) + \int_{\theta_d(a,b)}^{\infty} \bar{\pi}(a,b,\theta) d\Phi(\theta) \right\} \\
&= \alpha + \beta \left\{ \bar{\pi}(a,b) + \int_{\theta_d(a,b)}^{\infty} [\bar{\pi}(a,b,\theta) - \bar{\pi}(a,b)] d\Phi(\theta) \right\} \\
&= \alpha + \beta \Pi[a,b,\theta_d(a,b)],
\end{aligned}
$$

and using (14),

$$
U_a(a,b,s) = \beta \Pi_a[a,b,\theta_d(a,b)].
$$

Then, equation (19) implies

$$
\beta = \frac{c'(a_d^*)}{\Pi_a[a_d^*, b_d^*, \theta_d(a_d^*, b_d^*)]},
$$

and equation (20) implies

$$
\alpha = c(a_d^*) - \beta \Pi[a_d^*, b_d^*, \theta_d(a_d^*, b_d^*)].
$$

That is, we indeed have an optimal linear contract with $\beta > 0$.

Proof of Proposition 2

We have

$$\bar{\pi}_a(a,b,\theta_r) = h_a(a,b)\int \pi f_h[\pi|h(a,b),\theta_r]d\pi\,,$$
$$\bar{\pi}_b(a,b,\theta_r) = h_b(a,b)\int \pi f_h[\pi|h(a,b),\theta_r]d\pi\,, \tag{21}$$

and similarly for $\bar{\pi}_a(a,b)$ and $\bar{\pi}_b(a,b)$. We also have

$$\bar{g}_a(a,b,\theta_r) = \int_{\theta_c}^{\infty}[\bar{\pi}_a(a,b,\theta) - \bar{\pi}_a(a,b)]d\Phi(\theta),$$

$$\bar{g}_b(a,b,\theta_r) = \int_{\theta_c}^{\infty}[\bar{\pi}_b(a,b,\theta) - \bar{\pi}_b(a,b)]d\Phi(\theta).$$

Then, by (21), we have

$$\frac{\bar{g}_a(a,b,\theta_r)}{h_a(a,b)} = \frac{\bar{g}_b(a,b,\theta_r)}{h_b(a,b)}. \tag{22}$$

By (21), we also have

$$\frac{\bar{\pi}_a(a,b)}{h_a(a,b)} = \frac{\bar{\pi}_b(a,b)}{h_b(a,b)}.$$

Thus,

$$\frac{\Pi_a(a,b,\theta_r)}{h_a(a,b)} = \frac{\Pi_b(a,b,\theta_r)}{h_b(a,b)}. \tag{23}$$

By (6), we have

$$V_a(a,b,\theta_r,s) = \Pi_a(a,b,\theta_r) - U_a(a,b,\theta_r,s),$$
$$V_b(a,b,\theta_r,s) = \Pi_b(a,b,\theta_r) - U_b(a,b,\theta_r,s). \tag{24}$$

We also have

$$U_a(a,b,\theta_r,s) = h_a\int_0^{\theta_r}\int s(\pi)f_h[\pi|h(a,b)]d\pi d\Phi(\theta)$$
$$+h_a\int_{\theta_r}^{\infty}\int s(\pi)f_h[\pi|h(a,b),\theta]d\pi d\Phi(\theta) = \frac{h_a}{h_b}U_b(a,b,\theta_r,s). \tag{25}$$

Thus, by (23), (24) and (25),

$$V_b(a,b,\theta_r,s) = \Pi_b(a,b,\theta_r) - U_b(a,b,\theta_r,s) = \frac{h_b}{h_a}\Pi_a(a,b,\theta_r) - \frac{h_b}{h_a}U_a(a,b,\theta_r,s)$$
$$= \frac{h_b}{h_a}V_a(a,b,\theta_r,s). \tag{26}$$

Therefore, using (24) and (26), IC_a and IC_b in (7) imply

$$
\begin{aligned}
c'(a) &= U_a(a,b,\theta_r,s) = \Pi_a(a,b,\theta_r) - V_a(a,b,\theta_r,s) \\
&= \Pi_a(a,b,\theta_r) - \frac{h_a}{h_b} V_b(a,b,\theta_r,s) = \Pi_a(a,b,\theta_c) - \frac{h_a}{h_b} C'(b).
\end{aligned} \tag{27}
$$

This equation can replace one of the IC conditions. Also, since the IR condition must be binding, problem (7) becomes

$$
\begin{aligned}
&\max_{s\in\mathbb{S},a\in\mathbb{A},b\in\mathbb{B},\theta_r\geq 0} \Pi(a,b,\theta_r) - c(a) - C(b) \\
&\text{s.t.} \qquad \Pi_a(a,b,\theta_r) = c'(a) + \frac{h_a(a,b)}{h_b(a,b)} C'(b), \\
&\qquad\qquad U_a(a,b,\theta_r,s) = c'(a), \\
&\qquad\qquad U(a,b,\theta_r,s) = c(a).
\end{aligned}
$$

This problem can be solved in two steps. We first find the optimal efforts (a_r, b_r) and θ_r from the following problem:

$$
\begin{aligned}
&\max_{a\in\mathbb{A},b\in\mathbb{B},\theta_r\geq 0} \Pi(a,b,\theta_r) - c(a) - C(b) \\
&\text{s.t.} \qquad \Pi_a(a,b,\theta_r) = c'(a) + \frac{h_a(a,b)}{h_b(a,b)} C'(b).
\end{aligned} \tag{28}
$$

Then we find a salary package $s(\pi)$ that satisfies the following conditions:

$$
U_a(a,b,\theta_r,s) = c'(a), \tag{29}
$$

$$
U(a,b,\theta_r,s) = c(a). \tag{30}
$$

Given $(a_r^*, b_r^*, \theta_r^*)$ from problem (28), consider a linear scheme of the form $s(\pi) = \alpha + \beta\pi$. Then,

$$
\begin{aligned}
U(a,b,\theta_r,s) &= \alpha + \beta\left\{\int_0^{\theta_r}\int \pi f[\pi|h(a,b)]d\pi d\Phi(\theta) + \int_{\theta_r}^{\infty}\int \pi f[\pi|h(a,b),\theta]d\pi d\Phi(\theta)\right\}, \\
&= \alpha + \beta\left\{\int_0^{\theta_r}\bar{\pi}(a,b)d\Phi(\theta) + \int_{\theta_r}^{\infty}\bar{\pi}(a,b,\theta)d\Phi(\theta)\right\} \\
&= \alpha + \beta\left\{\bar{\pi}(a,b) + \int_{\theta_r}^{\infty}[\bar{\pi}(a,b,\theta) - \bar{\pi}(a,b)]d\Phi(\theta)\right\} \\
&= \alpha + \beta\left\{\bar{\pi}(a,b) + \int_{\theta_r}^{\infty}\bar{g}(a,b,\theta)d\Phi(\theta)\right\} = \alpha + \beta\Pi(a,b,\theta_r).
\end{aligned}
$$

Then,

$$
U_a(a,b,\theta_r,s) = \beta\Pi_a(a,b,\theta_r).
$$

Then, equation (29) implies

$$
\beta = \frac{c'(a_r^*)}{\Pi_a(a_r^*, b_r^*, \theta_c^*)},
$$

and equation (30) implies

$$
\alpha = c(a_r^*) - \beta\Pi(a_r^*, b_r^*, \theta_r^*).
$$

That is, we indeed have an optimal linear contract with $\beta > 0$.

Proof of Proposition 3

In the case of a discretionary contract, with verifiable efforts, the IC conditions are not needed. Without the two IC conditions, problem (4) becomes the following first-best problem:[5]

$$W_d^{**} \equiv \max_{s\in\mathbb{S}, a\in\mathbb{A}, b\in\mathbb{B}} \Pi[a,b,\theta_d(a,b)] - U(a,b,s) - C(b)$$
$$\text{s.t.} \quad U(a,b,s) \geq c(a).$$

Since the IR condition must be binding, the problem becomes

$$\begin{aligned} W_d^{**} \equiv \max_{s\in\mathbb{S}, a\in\mathbb{A}, b\in\mathbb{B}} & \Pi[a,b,\theta_n(a,b)] - c(a) - C(b) \\ \text{s.t.} \quad & U(a,b,s) = c(a). \end{aligned} \tag{31}$$

This problem can be solved in two steps. In the first step, we find the optimal efforts (a_d^{**}, b_d^{**}) by maximizing the objective function without the constraint. By (14), the first-best efforts (a_d^{**}, b_d^{**}) are determined by

$$\Pi_a[a_d^{**}, b_d^{**}, \theta_d(a_d^{**}, b_d^{**})] = c'(a_d^{**}), \qquad \Pi_b[a_d^{**}, b_d^{**}, \theta_d(a_d^{**}, b_d^{**})] = C'(b_d^{**}). \tag{32}$$

In the second step, given the efforts from (32), we consider a fixed salary package $s(\pi) = \beta$ for a constant β. It is a simple salary package that satisfies the constraint in (31), by which we find $\beta = \bar{u} + c(a_d^{**})$. That is, there is indeed a salary package that satisfies the constraint.

In the case of a rules-based contract, with verifiable efforts, the IC conditions are again not needed. Without the two IC conditions, problem (7) becomes the following first-best problem:

$$\begin{aligned} W_r^{**} \equiv \max_{s\in\mathbb{S}, a\in\mathbb{A}, b\in\mathbb{B}, \theta_r\geq 0} & \Pi(a,b,\theta_r) - U(a,b,\theta_r,s) - C(b) \\ \text{s.t.} \quad & U(a,b,\theta_r,s) \geq c(a). \end{aligned} \tag{33}$$

Since the IR condition must be binding, the problem becomes

$$W_r^{**} \equiv \max_{s\in\mathbb{S}, a\in\mathbb{A}, b\in\mathbb{B}, \theta_r\geq 0} \Pi(a,b,\theta_r) - c(a) - C(b)$$
$$\text{s.t.} \quad U(a,b,\theta_r,s) = c(a).$$

This problem can be solved in two steps. In the first step, we solve for optimal (a_r, b_r, θ_r) from the following problem

$$\max_{a\in\mathbb{A}, b\in\mathbb{B}, \theta\in\mathbb{R}_+} \Pi(a,b,\theta) - c(a) - C(b).$$

In particular, the first-order condition (FOC) for θ_r is

$$\Pi_\theta(a_r, b_r, \theta_r) = 0,$$

implying

$$g(a_r, b_r, \theta_r)\phi(\theta_r) = 0.$$

[5] In our terminology, an optimization problem is called the first-best problem if the efforts are verifiable.

With $\phi(\theta_r) > 0$, we have $g(a_r, b_r, \theta_r) = 0$, implying $\theta_r = \theta_d(a_r, b_r)$. Hence, the first-best efforts $(a_r^{**}, b_r^{**}, \theta_r^{**})$ are determined by

$$\begin{aligned} &\Pi_a(a_r^{**}, b_r^{**}, \theta_r^{**}) = c'(a_r^{**}), \\ &\Pi_b(a_r^{**}, b_r^{**}, \theta_r^{**}) = C'(b_r^{**}), \\ &\theta_r^{**} = \theta_d(a_r^{**}, b_r^{**}). \end{aligned} \tag{34}$$

In the second step, given $(a_r^{**}, b_r^{**}, \theta_r^{**})$, we try to find a salary package $s(\pi)$ that satisfies $U(a_r^{**}, b_r^{**}, \theta_r^{**}, s) = c(a_r^{**})$. Let $s(\pi) = \beta$, where β is a fixed constant. We find this fixed scheme to be $s_r^{**}(\pi) = c(a_r^{**})$.

Hence, by (34) and (14), the pair of efforts (a_r^{**}, b_r^{**}) for the first-best rules-based contract is the solution to the following problem:

$$W_r^{**} \equiv \max_{a \in \mathbb{A}, b \in \mathbb{B}} \Pi[a, b, \theta_d(a, b)] - c(a) - C(b). \tag{35}$$

In contrast, as implied by (31), the pair of efforts (a_d^{**}, b_d^{**}) for the first-best discretionary contract is the solution to the following problem:

$$W_d^{**} \equiv \max_{a \in \mathbb{A}, b \in \mathbb{B}} \Pi[a, b, \theta_d(a, b)] - c(a) - C(b). \tag{36}$$

Problems (35) and (36) are exactly the same. Hence, $(a_r^{**}, b_r^{**}) = (a_d^{**}, b_d^{**})$.

Proof of Proposition 4

Given the functions in (9), we have

$$\bar{\pi}(a, b) = h(a, b), \qquad \theta_d(a, b) = \frac{I}{h(a, b)}, \qquad g(a, b, \theta) = \theta h(a, b) - I,$$
$$\bar{g}(a, b, \theta) = \int_\theta^\infty [th(a, b) - I] d\Phi(t), \qquad \Pi(a, b, \theta) = h(a, b) + \bar{g}(a, b, \theta).$$

A General Solution

Consider the following problem:

$$\begin{aligned} W^* \equiv \max_{a \in \mathbb{A}, b \in \mathbb{B}} \; & \rho h(a, b) - \gamma_1 a - \gamma_2 b - \delta \\ \text{s.t.} \quad & \rho h(a, b) = \frac{\gamma_1}{\mu_1} a + \frac{\gamma_2}{\mu_2} b, \end{aligned}$$

where $\rho > 0$ and $\delta \geq 0$ are two constants. Given $h(a, b) = a^{\mu_1} b^{\mu_2}$, the solution to this problem is

$$a^* = \left[\rho\left(\frac{\mu_1^2(1+\Gamma)}{\gamma_1(\mu_1+\Gamma)}\right)^{1-\mu_2}\left(\frac{\mu_2^2(1+\Gamma)}{\gamma_2(\mu_2+\Gamma)}\right)^{\mu_2}\right]^{\frac{1}{1-\mu_1-\mu_2}},$$

$$b^* = \left[\rho\left(\frac{\mu_2^2(1+\Gamma)}{\gamma_2(\mu_2+\Gamma)}\right)^{1-\mu_1}\left(\frac{\mu_1^2(1+\Gamma)}{\gamma_1(\mu_1+\Gamma)}\right)^{\mu_1}\right]^{\frac{1}{1-\mu_1-\mu_2}}, \tag{37}$$

$$W^* = \rho^{\frac{1}{1-\mu_1-\mu_2}}\Lambda - \delta,$$

where

$$\Gamma \equiv \left(\sqrt{1+\frac{1-\mu_1-\mu_2}{\mu_1\mu_2}}-1\right)^{-1} = \left(\sqrt{\frac{(1-\mu_1)(1-\mu_2)}{\mu_1\mu_2}}-1\right)^{-1} = \left(\frac{1-\mu_1-\mu_2}{\sqrt{\mu_1\mu_2}\left(\sqrt{(1-\mu_1)(1-\mu_2)}+\sqrt{\mu_1\mu_2}\right)}\right)^{-1},$$

$$\Lambda \equiv \left(1-\frac{1+\Gamma}{\mu_1+\Gamma}\mu_1^2-\frac{1+\Gamma}{\mu_2+\Gamma}\mu_2^2\right)\left[\left(\frac{1+\Gamma}{\mu_1+\Gamma}\frac{\mu_1^2}{\gamma_1}\right)^{\mu_1}\left(\frac{1+\Gamma}{\mu_2+\Gamma}\frac{\mu_2^2}{\gamma_2}\right)^{\mu_2}\right]^{\frac{1}{1-\mu_1-\mu_2}}.$$

Notice that Γ and Λ do not depend on ρ, δ, p, I and θ_0. Notice also that we always have

$$\frac{1+\Gamma}{\mu_1+\Gamma}\mu_1^2+\frac{1+\Gamma}{\mu_2+\Gamma}\mu_2^2 < \mu_1+\mu_2 < 1,$$

implying $\Lambda > 0$. We have

$$h(a^*,b^*) = \left[\rho\left(\frac{\mu_1^2(1+\Gamma)}{\gamma_1(\mu_1+\Gamma)}\right)^{1-\mu_2}\left(\frac{\mu_2^2(1+\Gamma)}{\gamma_2(\mu_2+\Gamma)}\right)^{\mu_2}\right]^{\frac{\mu_1}{1-\mu_1-\mu_2}}\left[\rho\left(\frac{\mu_2^2(1+\Gamma)}{\gamma_2(\mu_2+\Gamma)}\right)^{1-\mu_1}\left(\frac{\mu_1^2(1+\Gamma)}{\gamma_1(\mu_1+\Gamma)}\right)^{\mu_1}\right]^{\frac{\mu_2}{1-\mu_1-\mu_2}}$$

$$= \rho^{\frac{\mu_1+\mu_2}{1-\mu_1-\mu_2}}\left(\frac{\mu_2^2(1+\Gamma)}{\gamma_2(\mu_2+\Gamma)}\right)^{\frac{\mu_2}{1-\mu_1-\mu_2}}\left(\frac{\mu_1^2(1+\Gamma)}{\gamma_1(\mu_1+\Gamma)}\right)^{\frac{\mu_1}{1-\mu_1-\mu_2}}$$

$$= \frac{\rho^{\frac{\mu_1+\mu_2}{1-\mu_1-\mu_2}}}{1-\frac{\mu_1^2(1+\Gamma)}{\mu_1+\Gamma}-\frac{\mu_2^2(1+\Gamma)}{\mu_2+\Gamma}}\Lambda.$$

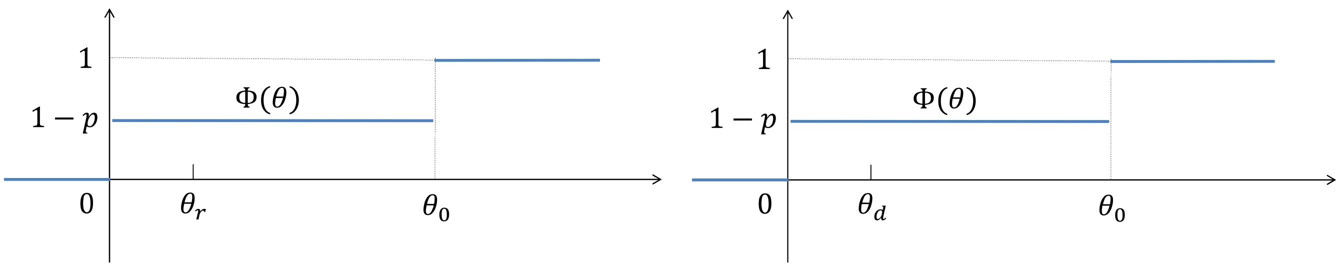

Figure 3. The Distribution Function $\Phi(\theta)$

Solutions with $\theta_r^* \le \theta_0$ and $\theta_r^* > \theta_0$

If $\theta_r > \theta_0$, then from Figure 3, we find

$$\bar{g}(a,b,\theta_r) = 0, \qquad \Pi(a,b,\theta_r) = h(a,b). \tag{38}$$

Then, problem (8) can be written as

$$\begin{aligned} W_{r+}^{*} \equiv & \max_{a\in\mathbb{A},b\in\mathbb{B}} h(a,b) - \gamma_1 a - \gamma_2 b \\ & \text{s.t.} \quad h(a,b) = \frac{\gamma_1}{\mu_1}a + \frac{\gamma_2}{\mu_2}b. \end{aligned} \tag{39}$$

If $\theta_r \le \theta_0$, then from Figure 3, we find

$$\begin{aligned} &\bar{g}(a,b,\theta_r) = p[\theta_0 h(a,b) - I] > 0, \\ &\Pi(a,b,\theta_r) = h(a,b) + p[\theta_0 h(a,b) - I] = (1 + p\theta_0)h(a,b) - pI. \end{aligned} \tag{40}$$

Then, problem (8) can be written as

$$\begin{aligned} W_{r-}^{*} \equiv & \max_{a\in\mathbb{A},b\in\mathbb{B}} (1+p\theta_0)h(a,b) - \gamma_1 a - \gamma_2 b - pI \\ & \text{s.t.} \quad (1+p\theta_0)h(a,b) = \frac{\gamma_1}{\mu_1}a + \frac{\gamma_2}{\mu_2}b. \end{aligned} \tag{41}$$

From problems (39) and (41), using (37), we find

$$\begin{aligned} &W_{r+}^{*} = \Lambda \\ &W_{r-}^{*} = (1+p\theta_0)^{\frac{1}{1-\mu_1-\mu_2}}\Lambda - pI. \end{aligned}$$

We have $W_{r+}^{*} > W_{r-}^{*}$ if and only if

$$I > \left[(1+p\theta_0)^{\frac{1}{1-\mu_1-\mu_2}} - 1\right]\frac{\Lambda}{p}. \tag{42}$$

Hence, $\theta_r^* > \theta_0$ if (42) holds; $\theta_r^* \le \theta_0$ if (42) fails. This solution is illustrated in Figure 4.

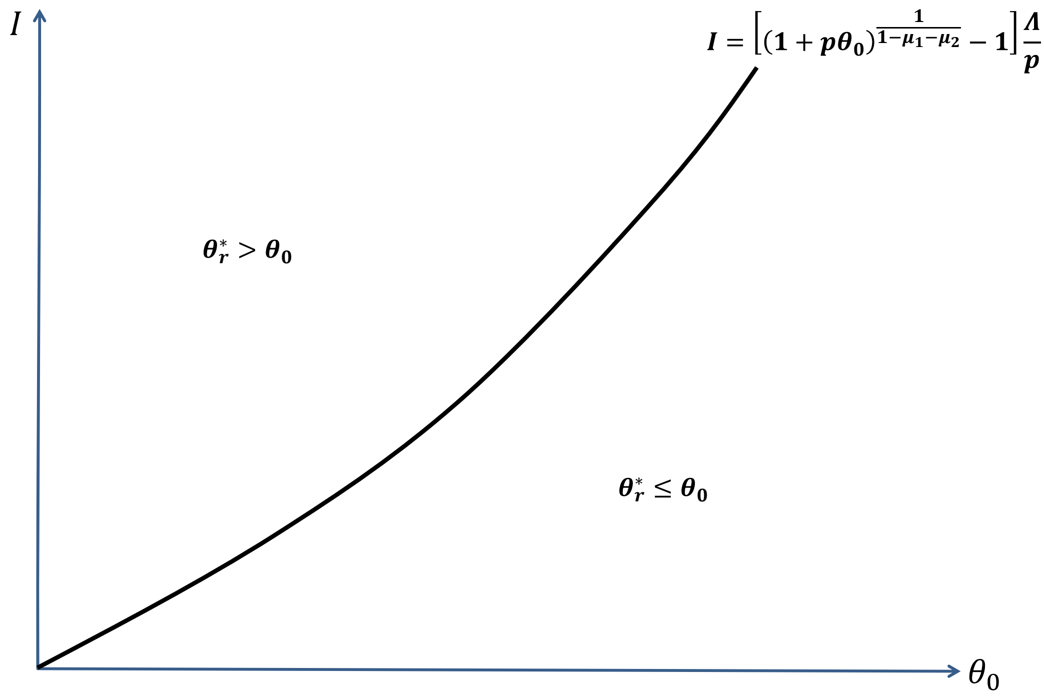

Figure 4. Determination of θ_r^*

Solutions with $\theta_d^* \leq \theta_0$ and $\theta_d^* > \theta_0$

If $\theta_d(a_d^*, b_d^*) > \theta_0$, then from Figure 3, we find

$$\bar{g}(a, b, \theta_d) = 0, \qquad \Pi(a, b, \theta_d) = h(a, b). \tag{43}$$

Then, problem (5) can be written as

$$\begin{aligned} W_{d+}^* \equiv \max_{a \in \mathbb{A}, b \in \mathbb{B}} & h(a, b) - \gamma_1 a - \gamma_2 b \\ \text{s.t.} \quad & h(a, b) = \frac{\gamma_1}{\mu_1} a + \frac{\gamma_2}{\mu_2} b. \end{aligned} \tag{44}$$

We denote by (a_{d+}^*, b_{d+}^*) the solution and $\theta_{d+} \equiv \theta_d(a_{d+}^*, b_{d+}^*)$ in this case since it is based on the assumption that $\theta_d(a_d^*, b_d^*) > \theta_0$. Since

$$\theta_d(a_d^*, b_d^*) = \frac{I}{h(a_d^*, b_d^*)},$$

the zones satisfying $\theta_{d+} > \theta_0$ are B and C in Figure 5, which are defined by

$$I > \theta_0 h(a_{d+}^*, b_{d+}^*).$$

This is illustrated in Figure 5, where

$$h(a_{d+}^*, b_{d+}^*) = \frac{\Lambda}{1 - \dfrac{\mu_1^2(1+\Gamma)}{\mu_1 + \Gamma} - \dfrac{\mu_2^2(1+\Gamma)}{\mu_2 + \Gamma}}.$$

If $\theta_d(a_d^*, b_d^*) \leq \theta_0$, then from Figure 3, we find

$$\begin{aligned} &\bar{g}(a, b, \theta_d) = p[\theta_0 h(a, b) - I] > 0, \\ &\Pi(a, b, \theta_d) = h(a, b) + p[\theta_0 h(a, b) - I] = (1 + p\theta_0) h(a, b) - pI. \end{aligned} \tag{45}$$

Then, problem (5) can be written as

$$\begin{aligned} W_{d-}^* \equiv \max_{a \in \mathbb{A}, b \in \mathbb{B}} & (1 + p\theta_0) h(a, b) - \gamma_1 a - \gamma_2 b - pI \\ \text{s.t.} \quad & (1 + p\theta_0) h(a, b) = \frac{\gamma_1}{\mu_1} a + \frac{\gamma_2}{\mu_2} b. \end{aligned} \tag{46}$$

We denote by (a_{d-}^*, b_{d-}^*) the solution and $\theta_{d-} \equiv \theta_d(a_{d-}^*, b_{d-}^*)$ in this case since it is based on the assumption that $\theta_d(a_d^*, b_d^*) \leq \theta_0$. Since

$$\theta_d(a_d^*, b_d^*) = \frac{I}{h(a_d^*, b_d^*)},$$

the zones satisfying $\theta_{d-} \leq \theta_0$ are A and B in Figure 5, which are defined by

$$I \leq \theta_0 h(a_{d-}^*, b_{d-}^*).$$

This is illustrated in Figure 5, where

$$h(a_{d-}^*, b_{d-}^*) = \frac{(1 + p\theta_0)^{\frac{\mu_1 + \mu_2}{1 - \mu_1 - \mu_2}}}{1 - \dfrac{\mu_1^2(1+\Gamma)}{\mu_1 + \Gamma} - \dfrac{\mu_2^2(1+\Gamma)}{\mu_2 + \Gamma}} \Lambda.$$

Given the above three zones, we now determine $\theta_d^* \equiv \theta_d(a_d^*, b_d^*)$ in each zone. In zone A, only the assumption that $\theta_d \leq \theta_0$ is consistent with a solution. There are only two possible assumptions, and the other assumption $\theta_d > \theta_0$ is not consistent with any solution in this zone. Hence, in zone A, $\theta_d^* = \theta_{d-}$.

In zone C, only the assumption that $\theta_d > \theta_0$ is consistent with a solution. Hence, in zone C, $\theta_d^* = \theta_{d+}$.

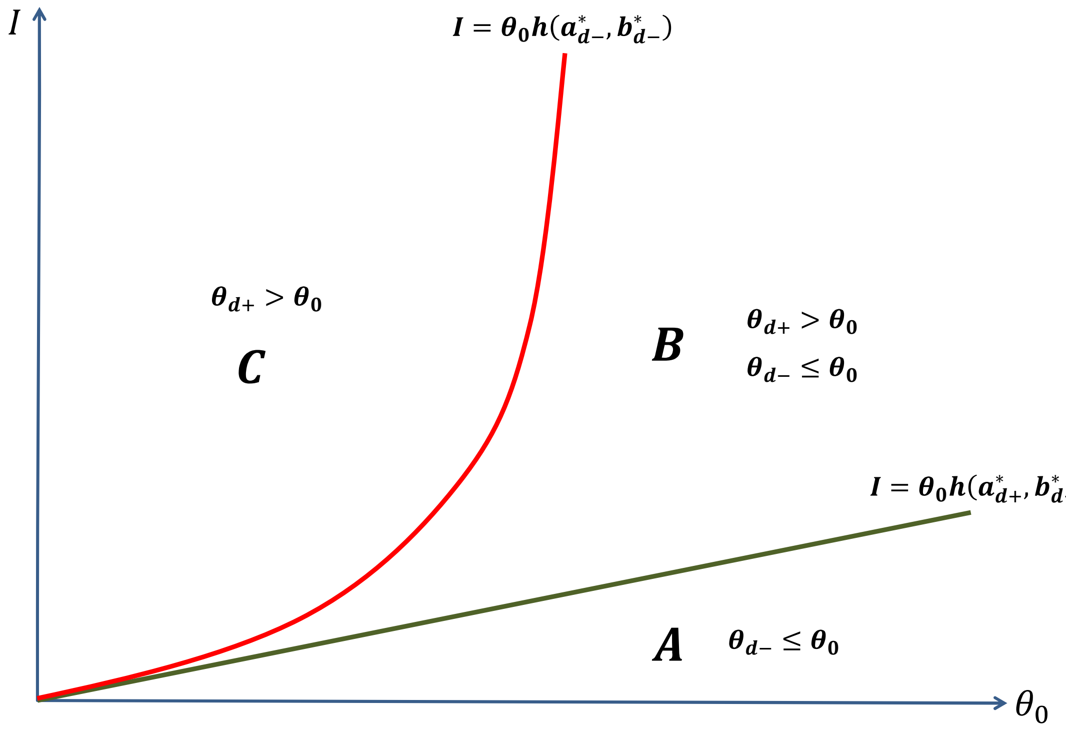

Figure 5. Determination of θ_d^*

In zone B, there are two possible assumptions $\theta_d > \theta_0$ and $\theta_d \leq \theta_0$, each being consistent with a solution. We need to compare these two solutions to determine the correct solution. Under the assumption that $\theta_d > \theta_0$, social welfare is

$$W_{d+}^* = \Lambda.$$

Under the assumption that $\theta_d \leq \theta_0$, social welfare is

$$W_{d-}^* = (1 + p\theta_0)^{\frac{1}{1-\mu_1-\mu_2}}\Lambda - pI.$$

We have $W_{d+}^* > W_{d-}^*$ if and only if

$$I > \left[(1 + p\theta_0)^{\frac{1}{1-\mu_1-\mu_2}} - 1\right]\frac{\Lambda}{p}. \tag{47}$$

Hence, $\theta_d^* = \theta_{d+}$ if (47) holds; $\theta_d^* = \theta_{d-}$ if (47) fails. This solution is illustrated in Figure 6.

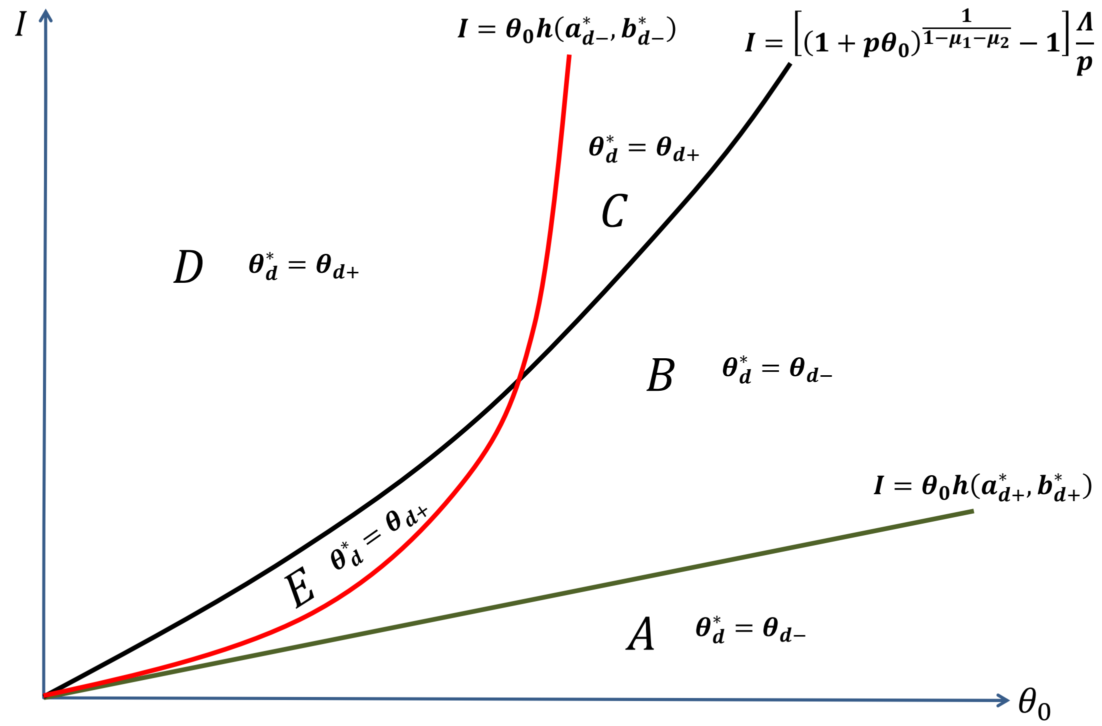

Figure 6. Determination of θ_d^*

Combining the above with the result on θ_r^*, we arrive at the following figure:

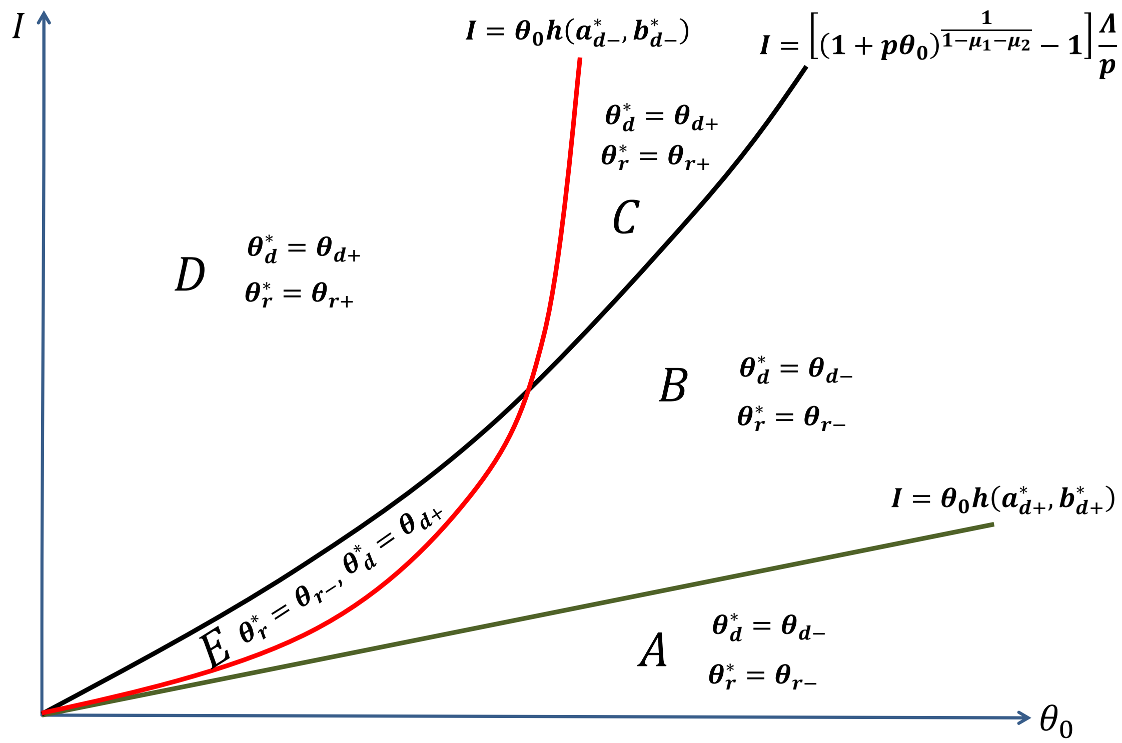

Figure 7. Zones for (θ_d^*, θ_r^*)

Zones A and B

Consider the case with $\theta_r \leq \theta_0$ and $\theta_d \leq \theta_0$. From problems (41) and (46), the two solutions are the same. Hence, $W_{r-}^* = W_{d-}^*$, implying that the two solutions are equally efficient.

Zones C and D

Consider the case with $\theta_r > \theta_0$ and $\theta_d > \theta_0$. From problems (39) and (44), the two solutions are the same. Hence, $W_{r+}^* = W_{d+}^*$, implying that the two solutions are equally efficient.

Zone E

Consider the case with $\theta_r \leq \theta_0$ and $\theta_d > \theta_0$. From problems (41) and (44), using (37), we find

$$W_{r-}^* = (1 + p\theta_0)^{\frac{1}{1-\mu_1-\mu_2}} \Lambda - pI$$
$$W_{d+}^* = \Lambda.$$

We have $W_{r-}^* \geq W_{d+}^*$ if and only if (42) fails. Hence, rules are more efficient than discretion in zone E. We hence have Figure 2.

Configuration of Figure 2

We now identify the relationships of the three curves in Figure 2. The curve defined by $I = h(a_{d+}^*, b_{d+}^*)\theta_0$ is a straight line and the curves defined by $I = h(a_{d-}^*, b_{d-}^*)\theta_0$ and $I = \left[(1 + p\theta_0)^{\frac{1}{1-\mu_1-\mu_2}} - 1\right]\frac{\Lambda}{p}$ are convex. All curves are upward sloping and start from the origin.

Since $h(a_{d-}^*, b_{d-}^*) > h(a_{d+}^*, b_{d+}^*)$, the curve defined by $I = h(a_{d-}^*, b_{d-}^*)\theta_0$ is above the curve defined by $I = h(a_{d+}^*, b_{d+}^*)\theta_0$.

To show that when θ_0 is small the curve defined by $I = h(a_{d-}^*, b_{d-}^*)\theta_0$ is below the curve defined by $I = \left[(1 + p\theta_0)^{\frac{1}{1-\mu_1-\mu_2}} - 1\right]\frac{\Lambda}{p}$, we need

$$\left.\frac{\partial[h(a_r^*, b_r^*)\theta_0]}{\partial\theta_0}\right|_{\theta_0=0} < \left.\frac{\partial\left[(1 + p\theta_0)^{\frac{1}{1-\mu_1-\mu_2}} - 1\right]\frac{\Lambda}{p}}{\partial\theta_0}\right|_{\theta_0=0},$$

or

$$\left.\frac{\partial}{\partial\theta_0}\left[\frac{\theta_0(1 + p\theta_0)^{\frac{\mu_1+\mu_2}{1-\mu_1-\mu_2}}}{1 - \frac{\mu_1^2(1+\Gamma)}{\mu_1+\Gamma} - \frac{\mu_2^2(1+\Gamma)}{\mu_2+\Gamma}}\Lambda\right]\right|_{\theta_0=0} < \left.\frac{\partial\left[(1 + p\theta_0)^{\frac{1}{1-\mu_1-\mu_2}} - 1\right]\frac{\Lambda}{p}}{\partial\theta_0}\right|_{\theta_0=0},$$

or

$$\left.\frac{(1+p\theta_0)^{\frac{\mu_1+\mu_2}{1-\mu_1-\mu_2}}+p\theta_0\frac{\mu_1+\mu_2}{1-\mu_1-\mu_2}(1+p\theta_0)^{\frac{2(\mu_1+\mu_2)-1}{1-\mu_1-\mu_2}}}{1-\frac{\mu_1^2(1+\Gamma)}{\mu_1+\Gamma}-\frac{\mu_2^2(1+\Gamma)}{\mu_2+\Gamma}}\right|_{\theta_0=0}<\frac{1}{1-\mu_1-\mu_2}(1+p\theta_0)^{\frac{\mu_1+\mu_2}{1-\mu_1-\mu_2}}\Big|_{\theta_0=0},$$

or

$$\frac{1}{1-\frac{\mu_1^2(1+\Gamma)}{\mu_1+\Gamma}-\frac{\mu_2^2(1+\Gamma)}{\mu_2+\Gamma}}<\frac{1}{1-\mu_1-\mu_2},$$

or

$$\frac{\mu_1^2(1+\Gamma)}{\mu_1+\Gamma}+\frac{\mu_2^2(1+\Gamma)}{\mu_2+\Gamma}<\mu_1+\mu_2,$$

which is true.

To show that when θ_0 is large the curve defined by $I=h(a_{d-}^*,b_{d-}^*)\theta_0$ is above the curve defined by $I=\left[(1+p\theta_0)^{\frac{1}{1-\mu_1-\mu_2}}-1\right]\frac{\Lambda}{p}$, for large θ_0, we need

$$\frac{\theta_0(1+p\theta_0)^{\frac{\mu_1+\mu_2}{1-\mu_1-\mu_2}}}{1-\frac{\mu_1^2(1+\Gamma)}{\mu_1+\Gamma}-\frac{\mu_2^2(1+\Gamma)}{\mu_2+\Gamma}}\Lambda>\left[(1+p\theta_0)^{\frac{1}{1-\mu_1-\mu_2}}-1\right]\frac{\Lambda}{p}. \tag{48}$$

For this, consider

$$\lim_{\theta_0\to\infty}\frac{\theta_0(1+p\theta_0)^{\frac{\mu_1+\mu_2}{1-\mu_1-\mu_2}}}{(1+p\theta_0)^{\frac{1}{1-\mu_1-\mu_2}}}=\lim_{\theta_0\to\infty}\frac{\theta_0}{1+p\theta_0}=\frac{1}{p}.$$

This implies that (48) holds when θ_0 is large enough. The relationships of the three curves are now clear.

Proof of Proposition 5

Zones A and B

In zones A and B, by (41) and (46),

$$\Pi(a,b,\theta_r)=\Pi(a,b,\theta_d)=(1+p\theta_0)h(a,b)-pI.$$

Since we also have $a_r^*=a_d^*$ and $b_r^*=b_d^*$ in these two zones, the two profit-sharing schemes are exactly the same in these two zones. According to Proposition 2, the income share β_r for the rules-based contract, which is the same as the income share β_d for the discretionary contract, is

$$\beta_r=\frac{c'(a_r^*)}{\Pi_a(a_r^*,b_r^*,\theta_c^*)}=\frac{\gamma_1}{(1+p\theta_0)h_a(a_r^*,b_r^*)}=\frac{\gamma_1 a_r^*}{(1+p\theta_0)\mu_1 h(a_r^*,b_r^*)}.$$

According to (37),

$$a_r^* = \left[(1+p\theta_0) \left(\frac{\mu_1^2(1+\Gamma)}{\gamma_1(\mu_1+\Gamma)} \right)^{1-\mu_2} \left(\frac{\mu_2^2(1+\Gamma)}{\gamma_2(\mu_2+\Gamma)} \right)^{\mu_2} \right]^{\frac{1}{1-\mu_1-\mu_2}},$$

$$h(a_r^*, b_r^*) = \frac{(1+p\theta_0)^{\frac{\mu_1+\mu_2}{1-\mu_1-\mu_2}}}{1 - \frac{\mu_1^2(1+\Gamma)}{\mu_1+\Gamma} - \frac{\mu_2^2(1+\Gamma)}{\mu_2+\Gamma}} \Lambda.$$

Hence,

$$\beta_r = \frac{\gamma_1 \left[\left(\frac{\mu_1^2(1+\Gamma)}{\gamma_1(\mu_1+\Gamma)} \right)^{1-\mu_2} \left(\frac{\mu_2^2(1+\Gamma)}{\gamma_2(\mu_2+\Gamma)} \right)^{\mu_2} \right]^{\frac{1}{1-\mu_1-\mu_2}}}{\frac{\mu_1 \Lambda}{1 - \frac{\mu_1^2(1+\Gamma)}{\mu_1+\Gamma} - \frac{\mu_2^2(1+\Gamma)}{\mu_2+\Gamma}}} = \frac{\gamma_1}{\mu_1} \frac{1+\Gamma}{\mu_1+\Gamma} \frac{\mu_1^2}{\gamma_1} = \frac{1+\Gamma}{\mu_1+\Gamma} \mu_1$$

$$= \frac{\sqrt{\mu_1\mu_2(1-\mu_1)(1-\mu_2)} + (1-\mu_1)(1-\mu_2)}{\sqrt{\mu_1\mu_2(1-\mu_1)(1-\mu_2)} + \mu_1(1-\mu_1)} \mu_1.$$

Zones C and D

In zones C and D, by (39) and (44),

$$\Pi(a, b, \theta_r) = \Pi(a, b, \theta_d) = h(a, b).$$

Since we also have $a_r^* = a_d^*$ and $b_r^* = b_d^*$ in these two zones, the two profit-sharing schemes are exactly the same in these two zones. According to Proposition 2, the income share β_r for the rules-based contract, which is the same as the income share β_d for the discretionary contract, is

$$\beta_r = \frac{c'(a_r^*)}{\Pi_a(a_r^*, b_r^*, \theta_c^*)} = \frac{\gamma_1}{h_a(a_r^*, b_r^*)} = \frac{\gamma_1 a_r^*}{\mu_1 h(a_r^*, b_r^*)}.$$

According to (37),

$$a_r^* = \left[\left(\frac{\mu_1^2(1+\Gamma)}{\gamma_1(\mu_1+\Gamma)} \right)^{1-\mu_2} \left(\frac{\mu_2^2(1+\Gamma)}{\gamma_2(\mu_2+\Gamma)} \right)^{\mu_2} \right]^{\frac{1}{1-\mu_1-\mu_2}},$$

$$h(a_r^*, b_r^*) = \frac{\Lambda}{1 - \frac{\mu_1^2(1+\Gamma)}{\mu_1+\Gamma} - \frac{\mu_2^2(1+\Gamma)}{\mu_2+\Gamma}}.$$

Hence,

$$\beta_r = \frac{\gamma_1 \left[\left(\frac{\mu_1^2(1+\Gamma)}{\gamma_1(\mu_1+\Gamma)} \right)^{1-\mu_2} \left(\frac{\mu_2^2(1+\Gamma)}{\gamma_2(\mu_2+\Gamma)} \right)^{\mu_2} \right]^{\frac{1}{1-\mu_1-\mu_2}}}{\frac{\mu_1 \Lambda}{1 - \frac{\mu_1^2(1+\Gamma)}{\mu_1+\Gamma} - \frac{\mu_2^2(1+\Gamma)}{\mu_2+\Gamma}}},$$

which is the same as that in zones A and B.

Zones E

In region E, by (41) and (44),

$$\Pi(a,b,\theta_r) = (1+p\theta_0)h(a,b) - pI,$$
$$\Pi(a,b,\theta_d) = h(a,b).$$

According to Proposition 1, the profit share for the discretionary contract is

$$\beta_d = \frac{c'(a_d^*)}{\Pi_a[a_d^*, b_d^*, \theta_d(a_d^*, b_d^*)]} = \frac{\gamma_1}{h_a(a_d^*, b_d^*)} = \frac{\gamma_1 a_d^*}{\mu_1 h(a_d^*, b_d^*)}.$$

According to (37),

$$a_d^* = \left[\left(\frac{\mu_1^2(1+\Gamma)}{\gamma_1(\mu_1+\Gamma)}\right)^{1-\mu_2}\left(\frac{\mu_2^2(1+\Gamma)}{\gamma_2(\mu_2+\Gamma)}\right)^{\mu_2}\right]^{\frac{1}{1-\mu_1-\mu_2}},$$
$$h(a_d^*, b_d^*) = \frac{\Lambda}{1-\frac{\mu_1^2(1+\Gamma)}{\mu_1+\Gamma}-\frac{\mu_2^2(1+\Gamma)}{\mu_2+\Gamma}}.$$

Hence,

$$\beta_d = \frac{\gamma_1\left[\left(\frac{\mu_1^2(1+\Gamma)}{\gamma_1(\mu_1+\Gamma)}\right)^{1-\mu_2}\left(\frac{\mu_2^2(1+\Gamma)}{\gamma_2(\mu_2+\Gamma)}\right)^{\mu_2}\right]^{\frac{1}{1-\mu_1-\mu_2}}}{\mu_1\left[\left(\frac{1+\Gamma}{\mu_1+\Gamma}\frac{\mu_1^2}{\gamma_1}\right)^{\mu_1}\left(\frac{1+\Gamma}{\mu_2+\Gamma}\frac{\mu_2^2}{\gamma_2}\right)^{\mu_2}\right]^{\frac{1}{1-\mu_1-\mu_2}}}.$$

We again find that $\beta_d = \beta_r$ and this profit share is the same as that in other zones.

References

Agarwal, V.; Daniel, N.D.; Naik, N.Y. (2015). The Role of Managerial Incentives and Discretion in Hedge Fund Performance. *Journal of Finance*, forthcoming.

Agarwal, V.; Daniel, N.D.; Naik, N.Y. (2009). Role of managerial incentives and discretion in hedge fund performance. *Journal of Finance*, 64(5), 2221-2256.

Aghion, P.; Tirole, J. (1997). Formal and Real Authority in Organizations. *Journal of Political Economy*, 105(1), 1-29.

Antonelli, M.A. (2003). Efficient Influence Activities with Endogenous Rent. Public Choice, 114 (1-2), 219-236.

Antonelli, M.A. (2014). Organizational Governance: Managerial Discretion, Automatic Rules or Ethics? Working Paper, Sapienza University.

Barnabas, N.; Mekoth, N. (2010). Autonomy, market orientation and performance in Indian retail banking. *Asia Pacific Journal of Marketing and Logistics*, 22(3), 330-350.

Bises, B. (2000). Exemption or taxation for profits of non-profits? An answer from a model incorporating managerial discretion. *Public Choice*, 104, 19-39.

Caza, A.J. (2011). Testing alternate predictions for the performance consequences of middle managers' discretion. *Human Resource Management*, 50(1), 9-28.

Chang, E.C.; Wong, S.M.L. (2003). Managerial discretion and firm performance in China's listed firms. Working Paper 1036, University of Hong Kong.

Crossan, K. (2005). The Theory of the Firm and Alternative Theories of Firm Behavior: A Critique. *International Journal of Applied Institutional Governance*, 1(1).

Evans, T. (2012). Organizational Rules and Discretion in Adult Social Work. *British Journal of Social Work*, 1-20.

Gammelgaard, J.; McDonald, F.; Stephan, A.; Tüselmann, H., Dörrenbächer, C. (2010). Performance Effects of Subsidiary Developments in Collaboration, and Decision Making Power. CIBEM Working Paper, Copenhagen Business School, Denmark.

Groves, T.; Hong, Y.; McMillan, J.; Naughton, B. (1994). Autonomy and incentives in Chinese state enterprises. *Quarterly Journal Economics*, 109(1), 183-209.

He, W.; Mukherjee, T.K.; Wei, P. (2009). Agency problems in tracking stock and minority carve-out decisions: explaining the discrepancy in short- and long-term performances. *Journal of Economics and Finance*, 33(1), 27-42.

Heinecke, P. (2011). *Success Factors of Regional Strategies for Multinational Corporations: Appropriate Degrees of Management Autonomy*. Springer, Berlin, Germany.

Spreitzer, G.; Porath, C. (2012). Creating Sustainable *Performance. Harvard Business Review*, Jan-Feb.

James, E. (1983). How nonprofits grow: a model. *Journal of Policy Analysis and Management*, 2, 350-365.

Kaarbøe, K.; Gooderham, P.N.; Nørreklit, H.; eds. (2013). *Managing in Dynamic Business Environments: Between Control and Autonomy*. Edward Elgar Publishing.

Khanchel, I. (2009). Le Rôle du Pouvoir Discrétionnaire du Dirigeant dans l'Amélioration de la Performance: Cas des Entreprises Tunisiennes. *Revue des Sciences de Gestion,* 237(238), 95-103.

Krug, J.A. (2009). Mergers and Acquisitions: Turmoil in Top Management *Teams. Harvard Business Publishing*, Dec.

Kuratko, D.F; Hornsby, J.S.; Covin, J.G. (2014). Diagnosing a firm's internal environment for corporate entrepreneurship. *Business Horizons,* 57, 37-47.

Kydland, F.E.; Prescott, E.C. (1977). Rules rather than discretion: the inconsistency of optimal plans. *Journal of Political Economy*, 85(3), 473-490.

Li, Y.; Zhao, X. (2004). Da gudong guquan, jingli zizhuquan yu gongsi jixiao (Ownership concentration, managerial discretion and company performance). *Zhongguo ruankexue (China Soft Science)*, 4, 86-92.

López-Navarro, M.Á.; Camisón-Zornoza, C. (2003). The effect of group composition and autonomy on the performance of joint ventures (JVs): an analysis based on Spanish export JVs. *International Business Review*, 12(1), 17-39.

Migué, J.-L.; Bélanger, G. (1974). Toward a general theory of managerial discretion. *Public Choice*, 17, 27-43.

Milgrom, P. (1988). Employment contracts, influence activities and efficient organization design. *Journal of Political Economy*, 96(1), 42-60.

Milgrom P.; Roberts J. (1992). *Economics, Organization and Management*. Prentice Hall, New Jersey.

Milgrom P.; Roberts J. (2009). Bargaining costs, influence costs and the organization of economic activity, in R. S. Kroszner and L. Putterman (eds.) *The economic nature of the firm: a reader.* Cambridge University Press, 143-155.

Schiff, J.; Weisbrod, B. (1991). Competition between for-profit and nonprofit organizations in commercial markets. *Annales de l'Economie Publique Sociale et Cooperative*, 4, 619-639.

Stano, M. (1976). Monopoly power, ownership control, and corporate performance. *Bell Journal of Economics*, 7(2), 672-679.

Venaik, S. (1999). A model of global marketing in multinational firms: an empirical investigation. PhD Thesis, University of Sydney and University of New South Wales.

Wang, S.; Zhu, T. (2005). Control Allocation, Revenue Sharing, and Joint Ownership. *International Economic Review*, 46(3), 895-915.

Williamson, O.E. (1963). Managerial discretion and business behavior. *American Economic Review*, 53, 1032-1057.

Williamson, O.E. (1964). *The Economics of Discretionary Behavior*, Prentice Hall, New York.

Xu, L.C.; Zhu, T.; Lin, Y. (2005). Politician control, agency problems and ownership reform: evidence from China. *Economics of Transition*, 13(1), 1-24.